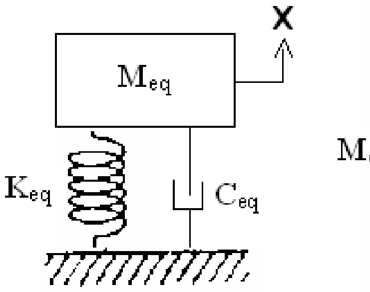
Damped Vibration

Topics:

- Introduction to Damped Vibration
- Damping Models
- Viscous Damping
- Energy Dissipation
- Damping Parameters
- Structural Damping
- Coulomb Damping
- Solution of Equations of Motion
- Logarithmic Decrement
- Practical Applications

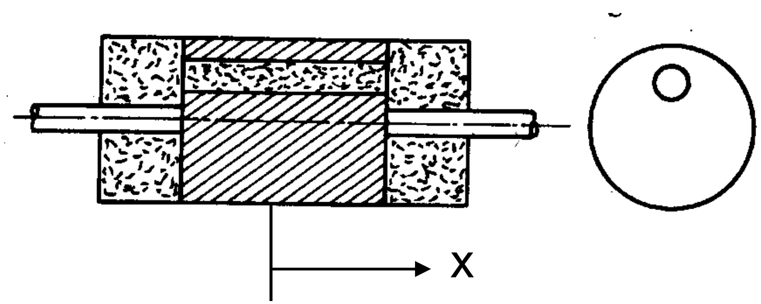
Introduction Damped Vibrations

1. Vibrating systems can encounter damping in various ways like
   - Intermolecular friction
   - Sliding friction
   - Fluid resistance

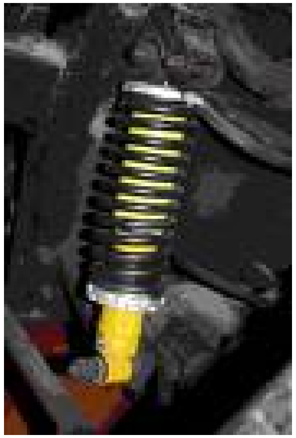


$$M_{eq}\ddot{x} + C_{eq}\dot{x} + K_{eq}x = 0$$

2. Damping estimation of any system is the most difficult process in any vibration analysis
3. The damping is generally complex and generally for mechanical systems it is so small to compute



Fluid resistant damper



Damping Models

Structural Damping	Coulomb Damping	Viscous Damping

Cause of Damping

Structural → Inherent Intermolecular friction

Coulomb → Rubbing between surfaces

Viscous → Fluid friction (viscosity)

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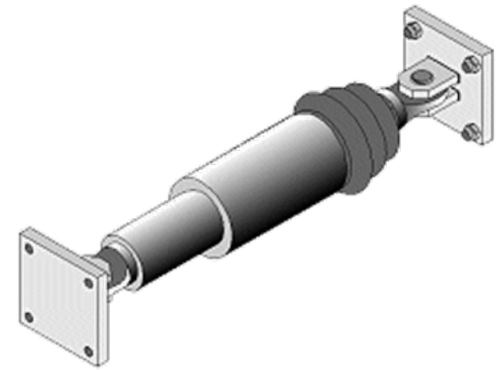
Viscous Damping

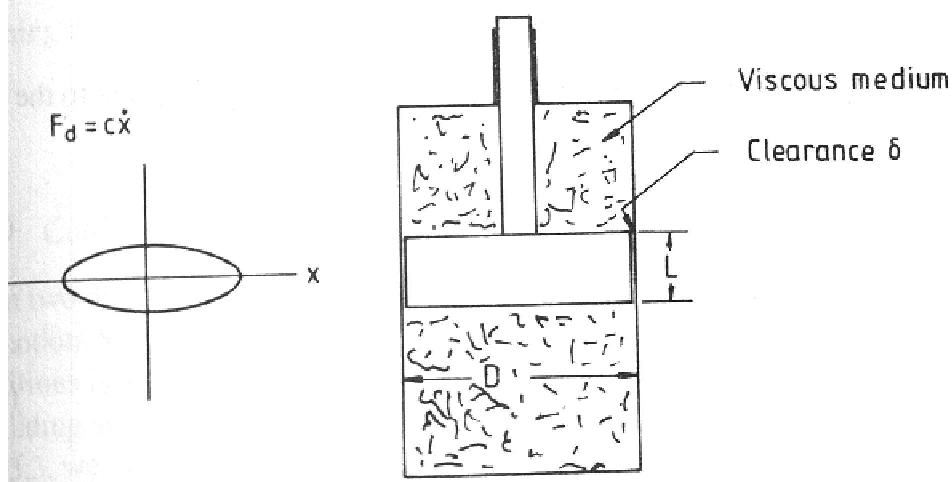
- Occurs when system vibrates in a viscous medium
- Obeys Newton's law of viscosity $\tau = \mu \dfrac{dv}{dz}$
- The damping force is given by $F_d = c\dot{x}$



c – damping constant

For the given viscous dashpot..

$$c = \frac{\pi D l \sigma}{\delta} \times 10^{-3}\ \text{Ns/m}$$



A simple viscous dashpot and its hysteresis loop

Energy Dissipation

Energy dissipation in case of of damping is extremely important because it gives the configuration of the damper

$$W_d = \int c\dot{x}dx$$ → Damping energy dissipated in viscous damping

Which comes out to be $W_d = \pi c w X^2$

So if we have any system and if we can calculate the energy dissipated by the system

The system can be modeled by the equivalent viscous damping as

$$C_{eq} = \frac{W_{dnv}}{\pi \omega X^2}$$

W_{dnv} is the energy dissipated due to non viscous damping

X – amplitude of vibration

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Damping parameters

The damping properties of the system can be compared using parameters like β and η

$$\beta = \frac{W_d}{V}$$ → Specific damping capacity

$$\eta = \frac{W_d}{2\pi V} = \frac{cw}{k}$$ → Loss coefficient

Specific damping capacity: Ratio of Energy loss per cycle to the max PE of system

Loss coefficient: Ratio of energy loss per radian to max PE of system

Hence using parameters like β and η we can compare two systems for which we don't know the value of c

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Structural Damping

Energy dissipation due to Inter Molecular Friction comes under structural damping category

Here energy dissipated is $W_d = \alpha X^2$

Hence equivalent damping is $C_{eq} = \dfrac{\alpha}{\pi w}$

$\alpha \longrightarrow$ Constant

Structural damping loss coefficient is given by $\Gamma = \dfrac{\alpha}{\pi k}$

Coulomb Damping

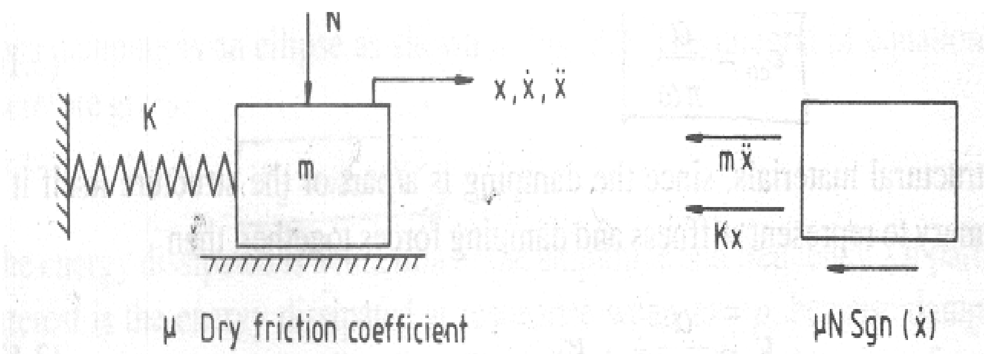
Equation of motion for coulomb damping is

$$m\ddot{x} + K(x \pm \Delta) = 0$$

$$m\ddot{x} + Kx = \mp K\Delta$$

Also $\mu N = K\Delta$

Opposite sign to be used for + and – ve velocity



Free body diagram

Coulomb Damping

For initial condition of x_0 displacement and zero input velocity

Solution is given by

$$x = \mp \Delta + (x_0 \pm \Delta) \cos pt$$

$$\dot{x} = -(x_0 \pm \Delta)\, p \sin pt$$

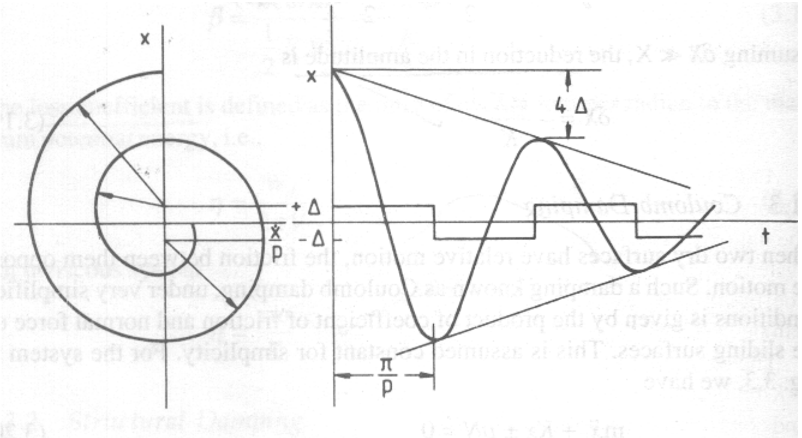


Fig. 3.4 Phase plane solution of Coulomb damping system

The decrease of 4Δ is very important is important characteristics of coulomb damping

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Solution of Equations of Motion

Now let us know how the solution of the equations of motion are effected by the introduction of damping parameters.

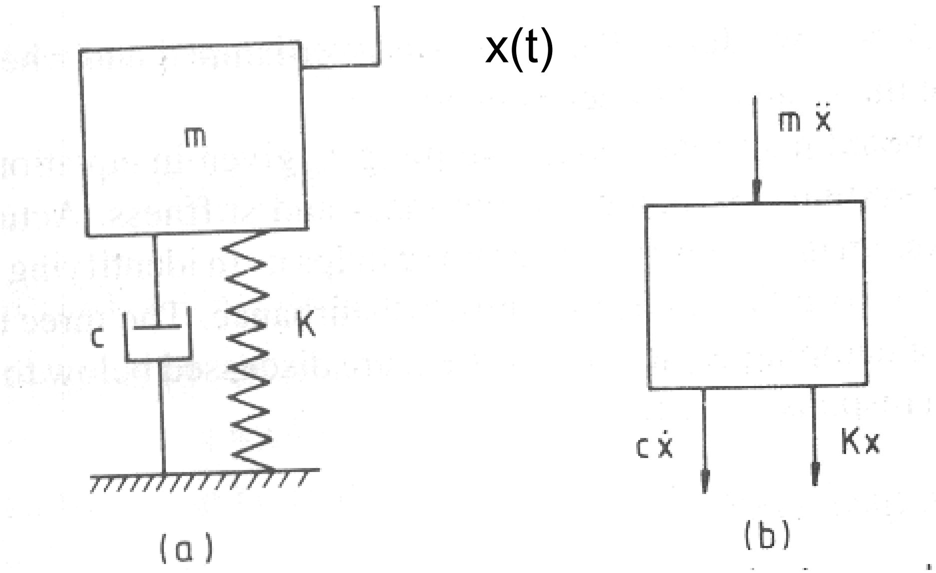
The generalized equation of motion is $M\ddot{x} + c\dot{x} + kx = 0$

The viscous damping is more common or in other terms equivalent viscous damping is more commonly used in place.

Hence using that in the analysis…

Solution of Equations of Motion

SDF viscous damping solutions



Damped SDF and free body diagram

The eqn of motion is given by

$$m\ddot{x} + c\dot{x} + Kx = 0$$

A particular solution is given by $x = e^{st}$

Hence $ms^2 + cs + K = 0$

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Solution of Equations of Motion

The two roots of the above equation are given by

$$s_{1,2} = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{K}{m}}$$

A general solution is given by

$$x = Ae^{s_1 t} + Be^{s_2 t}$$

To make the solution more general the critical damping coefficient is defined as

$$\left(\frac{c_c}{2m}\right)^2 = \frac{K}{m} = p^2$$

$$c_c = 2mp = 2\sqrt{Km}$$

which gives the criteria for various damping properties

where $\xi = \frac{c}{c_c}$

So the solution now becomes in terms of ξ as

$$s_{1,2} = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)p$$

This gives rise to four cases as discussed next

Solution of Equations of Motion

Type of System depends on Damping Factor

1. Under damped ξ <1
2. Undamped ξ =0
3. Over damped ξ >1
4. Critically damped ξ =1

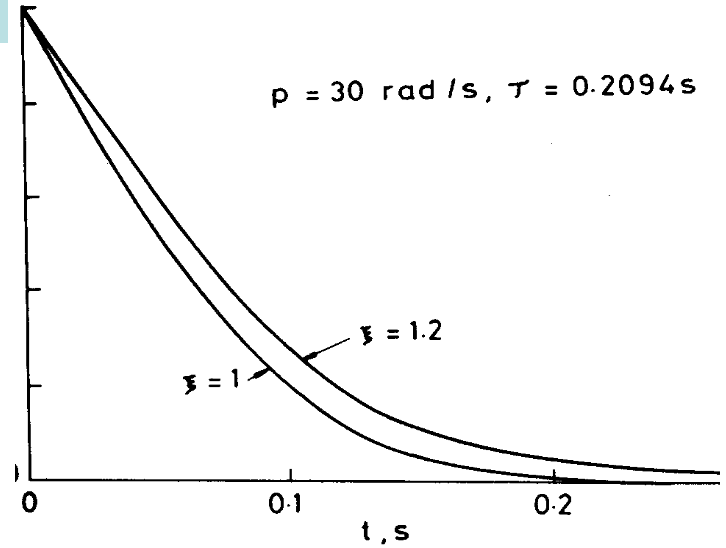


Fig. 3.6 Response of an overdamped system

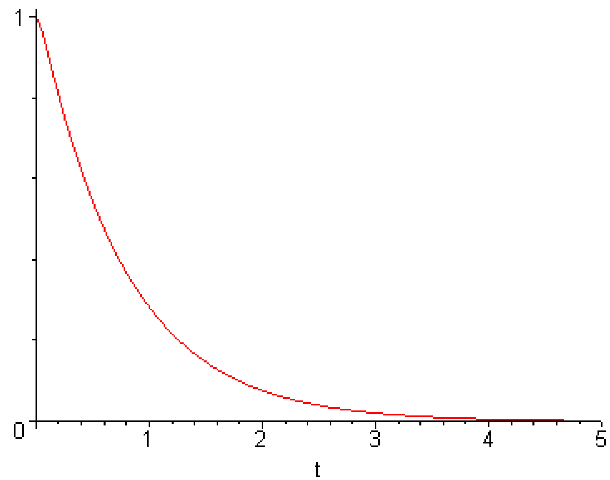
Over damped

The general solution becomes

$$x = Ae^{\left(-\xi + \sqrt{\xi^2 - 1}\right)pt} + Be^{\left(-\xi - \sqrt{\xi^2 - 1}\right)pt}$$

The solution will be non oscillatory and gradually comes to rest

Solution of Equations of Motion



Door Damper

Critically damped

The general solution becomes

$$x = (A + Bt)\, e^{-pt}$$

For zero initial conditions →

$$\frac{x}{x_0} = (1 + pt)\, e^{-pt}$$

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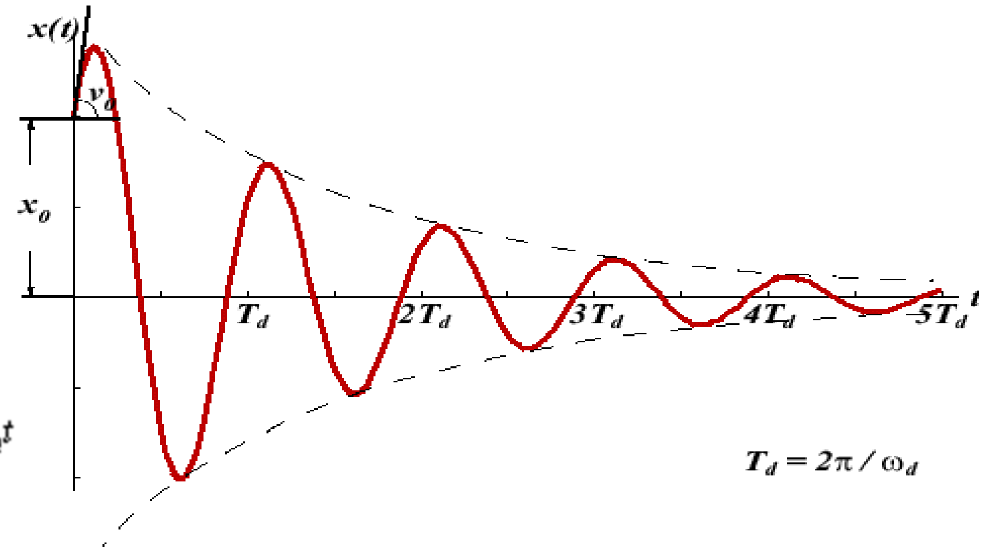
Solution of Equations of Motion

Under damped

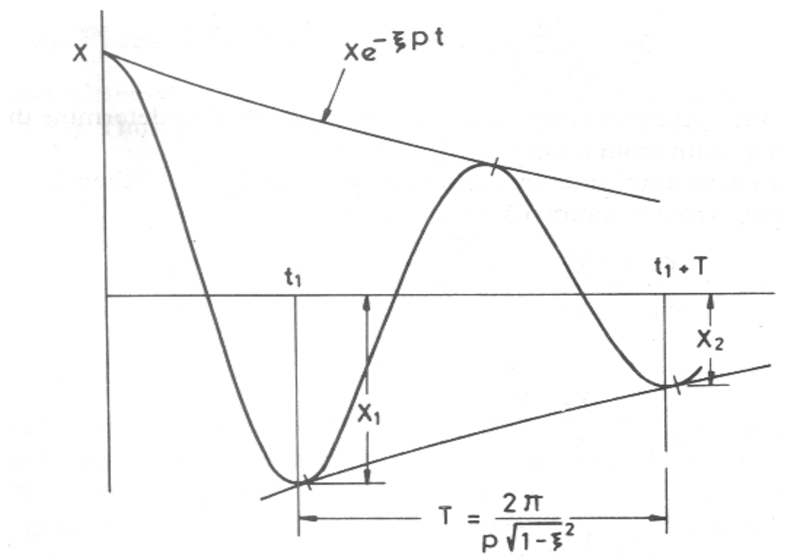


The general solution becomes

$$x(t) = c_1 e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + c_2 e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$$

The damped time period is given by

$$T_d = \frac{2\pi}{p\sqrt{1-\xi^2}}$$



The envelope for the under damped curve is given by $e^{-\xi\omega_n t}$ as shown

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Logarithmic Decrement

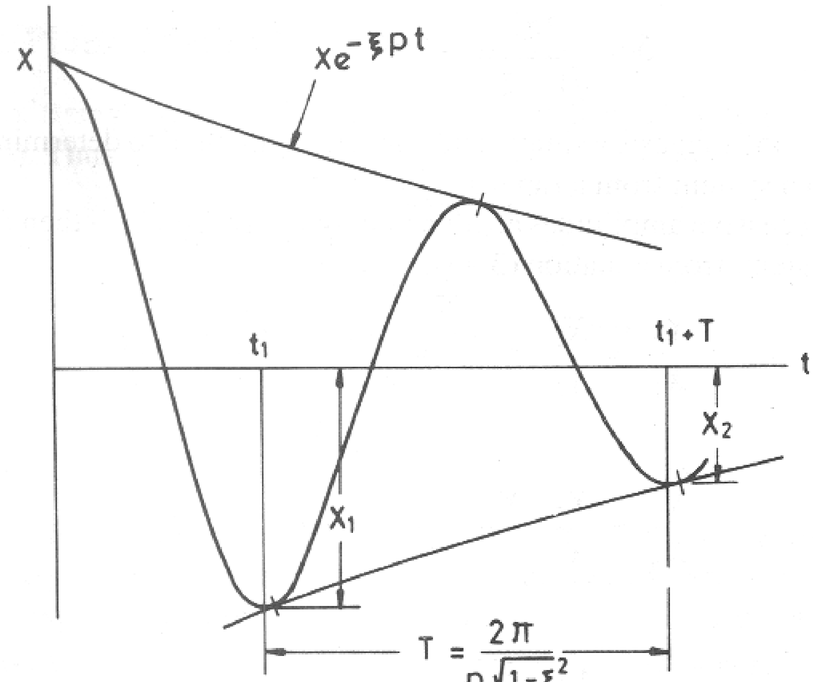
The logarithmic decrement is the Natural log of Ratio of the Successive Drops in the Amplitude

It is used for the estimation of the value of ξ of the system experimentally

For two points

$$X_1 = \exp(-\xi p t_1)$$
$$X_2 = \exp[-\xi p(t_1 + T)].$$

Since the outer envelope is an exponential curve



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Logarithmic Decrement

Logarithmic decrement is given by $\delta = \ln \frac{X_1}{X_2}$

That is $\delta = \ln \frac{\exp(-\xi p t_1)}{\exp[-\xi p(t_1 + T)]}$

$$= \xi p T$$

Which gives $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$

If we consider the drop in amplitude in n successive cycles then the log decrement is given by

$$\delta = \frac{1}{n} \ln \frac{X_0}{X_n}$$

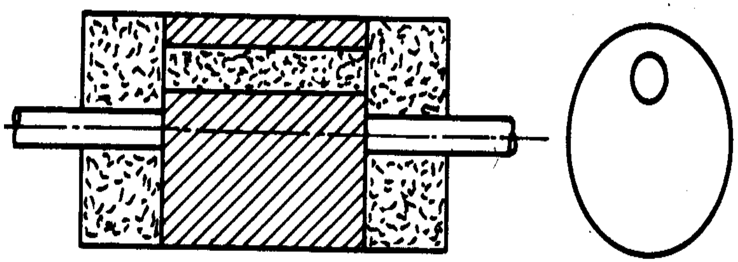
Practical Applications

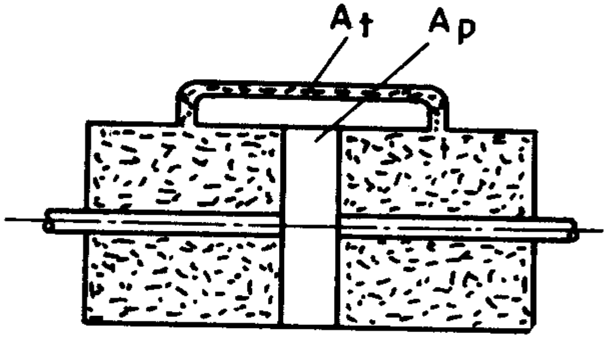
Viscous dampers:

Various types of viscous dampers are shown


Door Damper

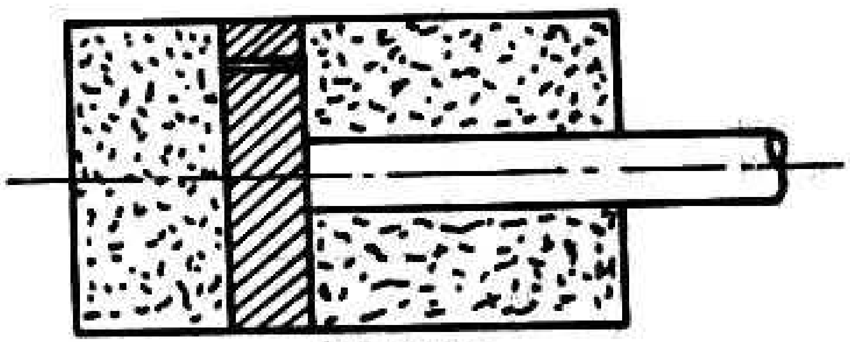

Gun Barrel





Assignment

1 Figure shows a piston with an orifice, placed in a cylinder filled with liquid. As the piston moves, the fluid displaced flows back through the orifice. The damping force provided by such an arrangement is known to be proportional to the square of velocity. Show that the equivalent viscous damping is dependent on amplitude as well as the frequency of vibration.



2 An automobile can be modelled as a mass placed on four shock absorbers (shock absorber consists of a spring and a damper), such that each spring is equally loaded. Determine stiffness and the damping constant of each shock absorber so that the natural frequency is 1.5 Hz and the system is critically damped. The mass of the vehicle is 1500 kg.

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Assignment

3 A machine of mass 2000kg mounted on isolators has a frequency of 8Hz. Loss modulus of the isolator in shear is found to be a function of the isolator strain amplitude. What will be the ratio of successive amplitudes in a free vibration rap test when peak to peak isolator strain amplitude is a) 0.001 and b) 0.05.